# Unified Framework of Emergent Recursion and the Infinite Whole — Rigorous Edition (v7, Unicode)

#### **Authors and Credits**

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Attribution boundary: Ford-credited sections include relational growth, dialog-derived principles, energy dual flow & cyclic renewal, and the new layer-complementarity & spin formalism. Barbee-Saunders is credited solely for the emergence field E, recursion operator  $D\alpha$ , octonionic bookkeeping O, and Big-Bang-compatible modeling mentioned in the comparative section.

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#### **Abstract**

We axiomatize a triad grammar (0, 1, mix), prove a dimensional-closure result (D = 3 by minimal uniqueness of a normal), and couple the grammar to a layered emergence formalism. This v7 adds a Ford-credited, fully worded formalism for layer complementarity: each layer concurrently exhibits capacity (0) and distinction (1) as complementary densities whose alternation produces a phase (spin). Spin drives absorption/redistribution between dual measures ("light"/"dark"), yielding continuous mixing (input/output) across opposite scales. Under isotropy, the global effect is a spherical, centerless, and relationally constant infinitum.

#### Table of Contents (selected)

- 1. 1. Notation and Preliminaries
- 2. 2. Axioms (Triad Grammar)
- 3. 3. Dimensional-Closure Theorem (Sketch)
- 4. 4. Emergence Field and Octonionic Law (Definitions) PSB-credited elements
- 5. 5. Coupling Map and Conserved Quantities
- 6. 6. Space-Time Orientation and Arrow of Becoming
- 7. 11. Relational Growth (Ford)
- 8. 12. Comparative Perspectives (Ford + PSB)
- 9. 13. Dialog-Derived Principles (Ford)
- 10. 14. Energy Dual Flow & Cyclic Renewal (Ford)
- 11. 15. Layer Complementarity, Spin, and Spherical Infinitum (Ford) New
- 12. Appendix A. Minimal Calculus on Recursion
- 13. Appendix B. Provenance and Citations

#### 1. Notation and Preliminaries

We use real inner-product spaces with basis  $e_0$ ,  $e_1$ ,  $e_2$ ; exterior product  $\Lambda$ ; and Hodge dual  $\bigstar$  in 3D. Recursion index  $n \in \mathbb{N}$ . Constants:  $\alpha = 7/11$ ,  $\varphi = (1+\sqrt{5})/2$ . Define a recursion derivative  $D\alpha$  and a layer Hamiltonian  $H_n = (\alpha^n) \cdot E_0$ .

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W=e_0 \Lambda e_1 ; J_{\bigstar}={_{\bigstar}(W)} ; K(x)=D\alpha^2x ; I_n=\alpha\cdot I_{n-1}+\delta_n ; q=D\alpha F ; m_n \propto \alpha^{*}(-n) .
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### 2. Axioms (Triad Grammar)

A1 Capacity (0). A2 Distinction (1). A3 Mixing. A4 Minimality. A5 Self-verification.

Persistent consequence of interaction:  $J\star=\star(e_0\ \Lambda\ e_1)$ , the reconciliation current.

### 3. Dimensional-Closure Theorem (Sketch)

Uniqueness of a carrier normal to any 2-surface of interaction enforces D=3: in D dimensions the unit normals form  $S^(D-3)$ , hence uniqueness  $\Rightarrow D-3=0$ .

# 4. Emergence Field and Octonionic Law (Definitions) — PSB-credited elements

Layered states  $\{\psi_n\}$ ; operators  $(D\alpha, T\phi)$ ; evolution  $i\cdot\hbar\psi\cdot\partial\psi_n/\partial t=H_n\cdot\psi_n$ ; octonionic domains O for triality bookkeeping. These items are credited to Barbee-Saunders.

## 5. Coupling Map and Conserved Quantities

 $M(capacity) \rightarrow baseline of E; M(distinction) \rightarrow excitation; M(mix) \rightarrow apply D\alpha, T\phi;$  reconciliation axis selects orientation. Coarse-grained conservation:  $d/dt(J\star,J\star) \geq 0$ .

## 6. Space-Time Orientation and Arrow of Becoming

The sign of  $J \star$  fixes an orientation that coarse-grains to an arrow of becoming (time).

#### 11. Relational Growth (Ford)

Absolute size is undefined for the whole; only ratios are physical. With  $D(t)=a(t)\cdot x$  and co-scaled units  $u(t) \propto a(t)$ , D(t)/u(t)=x. Expansion is a metric change without an external center; a no-boundary whole has zero net reconciliation flux.

#### 12. Comparative Perspectives (Ford + PSB)

Ford: eternal, centerless recursion; relational global constancy. PSB: Big-Bang-compatible expansion via emergence machinery E, D $\alpha$ , O. Unified view: hot starts appear as local high-excitation patches within an infinite whole.

### 13. Dialog-Derived Principles (Ford)

Present-as-0; Information-as-1; co-creation of time/space; layer genesis by rebalancing; mutual containment (1 in 0, 0 in 1); self-similar infinity; coexistence principle; round-canon analogy.

#### 14. Energy Dual Flow & Cyclic Renewal (Ford)

Useful work writes durable distinctions (information) while consuming free energy. The cosmos drains while it fills; cyclic renewal: erase  $\rightarrow$  capacity  $\rightarrow$  rewrite. Isotropic reconciliation supports a sphere-like, centerless whole; orbits are balance loci of opposing currents.

## 15. Layer Complementarity, Spin, and Spherical Infinitum (Ford) — New

- 15.1 Definitions (complementary densities). On layer n let  $C_n(x,t) \ge 0$  denote capacity density (0-like) and  $I_n(x,t) \ge 0$  denote information density (1-like). Define the complex order parameter  $\Psi_n(x,t) = \sqrt{I_n(x,t)} \cdot \exp[i \cdot \theta_n(x,t)]$ , where  $\theta_n$  is a phase. Complementarity is encoded by a parity operator  $\square$  with  $\square[C_n] = I_n$  and  $\square[I_n] = C_n$  at a shifted phase  $\theta_n \to \theta_n + \pi$ .
- 15.2 Dual continuity (absorption/redistribution). Introduce dual currents  $J_n$  (information) and  $K_n$  (capacity) with continuity laws:

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\partial_t I_n + \nabla \cdot J_n = + \kappa_n \cdot C_n - \mu_n \cdot I_n + \Sigma_n (write/erase and cross-layer terms) \partial_t C_n + \nabla \cdot K_n = - \kappa_n \cdot C_n + \mu_n \cdot I_n - \Sigma_n (complementary balance)
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Here  $\kappa_n$ ,  $\mu_n \ge 0$  are conversion coefficients;  $\Sigma_n$  accounts for exchange with adjacent layers. These equations formalize continuous absorption ("dark" uptake into capacity) and redistribution ("light" expression as information).

- 15.3 Spin as phase from alternation. Alternation between  $C_n$  and  $I_n$  induces a phase dynamics  $\partial_t \theta_n = \omega_n \gamma_n \cdot \Delta \theta_n + \xi_n$ , where  $\omega_n \propto \alpha^n$  is the natural frequency,  $\gamma_n \ge 0$  is a smoothing coefficient, and  $\xi_n$  collects couplings. The layer spin density is  $\sigma_n = \partial_t \theta_n$  and the associated vorticity is  $\Omega_n = \nabla \times (\nabla \theta_n)$ . Non-zero  $\Omega_n$  implies persistent rotational structure ("spin") at that layer.
- 15.4 Mixing across opposite scales. Let  $S_+$  denote outward (coarse) scales and  $S_-$  inward (fine) scales. Conservation under parity implies flux balance  $J_n(S_+) \approx -K_n(S_-)$  at stationarity, producing a standing exchange that sustains  $\sigma_n \neq 0$ . This is the formal statement of input/output mixing driving spin at every layer.
- 15.5 Spherical, centerless infinitum. If the distribution of layer phases  $\theta_n$  is isotropic on large scales, the average of  $\Omega_n$  over orientations vanishes while the scalar spin power  $\langle |\sigma_n|^2 \rangle$  remains finite. Isotropy at all points enforces a sphere-like (no preferred direction) metric in 3D, consistent with the minimal closure proof. Thus the whole is spherical in the statistical-geometric sense, centerless, and relationally constant while layers keep mixing.
- 15.6 Observable implications. (i) Phase-coupled oscillations between compressive and expressive modes in self-organizing media; (ii) cross-scale anti-correlations of dual fluxes (J<sub>n</sub> vs. K<sub>n</sub>); (iii) persistent vortex-like features with scale-dependent  $\omega_n \propto \alpha^n$ ; (iv) maintenance of dimensionless ratios despite net metric expansion.

## Appendix A. Minimal Calculus on Recursion

 $\label{eq:defDalpha} D\alpha \ f(x) \ = \ lim_{\{\epsilon \to 0\}} \ (\ f(x+\epsilon\alpha) \ - \ f(x) \ ) \ / \ \epsilon \ ; \ \psi_{\text{\tiny N+1}} \ = \ exp(-iH\Delta t/\hbar\psi) \ \psi_{\text{\tiny N}} \ .$ 

### Appendix B. Provenance and Citations

Ford-credited contributions: §§11, 13, 14, 15. PSB-credited elements: emergence field E, D $\alpha$ , octonionic bookkeeping O, and Big-Bang-compatible modeling in the comparative section. This edition embeds Unicode fonts to ensure all scientific symbols render correctly.